Winter term 2024/25
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Presence sheet 06

Mathematics for Machine Learning

Tutorial of **Week 07 (25.11. - 29.11.2024)**

Exercise 1 (Derivatives).

a) Calculate the gradient of $f : \mathbb{R}^3 \to \mathbb{R}$ with $f(x) = x_1 x_2^2 + x_1 x_2 x_3 + e^{x_1}$.

b) Calculate the Jacobian matrix of $f : \mathbb{R}^3 \to \mathbb{R}^2$ with $f(x) = \begin{pmatrix} \sin(x_1) \\ x_2^2 x_3^4 \end{pmatrix}$.

c) Calculate the total differentials for a) and b).

Exercise 2 (Matrix cookbook).

a) Use the matrix cookbook of the lecture to calculate the gradient of $f : \mathbb{R}^n \to \mathbb{R}$ with $f(x) = x^t Ax + b^t x + c$ for $b \in \mathbb{R}^n, c \in \mathbb{R}$ and $A \in \mathbb{R}^{n\times n}$ symmetric.

b) Show that for $f : \mathbb{R}^{n\times n} \to \mathbb{R}$ with $f(X) = \mathrm{tr}(X)$ we have $\frac{\partial f(X)}{\partial X} = I_n$.

Exercise 3 (Higher order derivatives).

Consider $f : \mathbb{R}^3 \to \mathbb{R}$ with $f(x) = 5x_1^3 x_2 + 2\cos(x_3)$ for $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$.

a) Calculate the gradient.

b) Calculate the Hessian matrix.

c) Find all critical points of f.

d) Decide for each of the critical points whether they are local/global minima/maxima or saddlepoints.

e) Why is the Hessian matrix symmetric?

Exercise 4 (Directional derivatives).

Consider $f : \mathbb{R}^3 \to \mathbb{R}$ with $f(x) = x_1^2 x_2 + 2\cos(x_3)$ for $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$.

a) Compute the directional derivative of f at point $p = \begin{pmatrix} 1 \\ 1 \\ \pi/2 \end{pmatrix}$ into the direction of vector $v = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

b) Determine the direction in which f increases most rapidly at point p.