Winter term 2024/25
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Presence sheet 13

Mathematics for Machine Learning

Tutorial of **Week 14 (27.01. - 31.01.2025)**

Exercise 1 (Maximum Likelihood Estimator).
Suppose you toss a coin $n = 10$ times and get head for $k = 7$ times. Now you want to estimate the probability of "head" p.

a) State the statistical model.

b) Calculate the Maximum Likelihood Estimator $\hat{p}_{MLE}$.

Exercise 2 (Multiple choice questions).
For each of the following questions, choose exactly one answer.

a) When testing the null hypothesis H_0 against H_1, the p-value of an observation is

   A. $P(H_0 \text{ is true})$

   B. $1 - P(H_1 \text{ is true})$

   C. the smallest level of a test, which would reject H_0

b) Consider two different tests of level α with power functions $\beta_1(\theta) = \theta^5$ (Test 1) and $\beta_2(\theta) = \theta^3$ (Test 2) for $\theta \in \Theta_1 = [1/2, 1)$. Which test would you prefer?

   A. Test 1 B. Test 2

c) Which of the following (95%)-confidence intervals for a parameter $\theta \in \mathbb{R}$ would you prefer?

   A. $I_1 = [-2, 3]$ B. $I_2 = [-1, 0]$ C. $I_3 = [-1, 3]$ D. $I_4 = [-2, 0]$

d) There always exists a test of level $\alpha = 0$.

   A. True B. False

e) A manufacturer claims that at least 99% of their produced cellphones are flawless. To test this statement, we observe independent samples $x_1, \ldots, x_n$ from a Bernoulli distribution $\mathrm{Ber}(\theta)$ for $\theta \in (0, 1)$, where $x_i = 1$ indicates that cellphone i is faulty and $x_i = 0$ indicates that it is flawless. How do we have to formulate a hypothesis test, if we want to control the error of wrongfully accusing the manufacturer of a false statement?

   A. $H_0 : \theta \geq 0.99$ and $H_1 : \theta < 0.99$

   B. $H_0 : \theta < 0.99$ and $H_1 : \theta \geq 0.99$

   C. $H_0 : \theta > 0.01$ and $H_1 : \theta \leq 0.01$

   D. $H_0 : \theta \leq 0.01$ and $H_1 : \theta > 0.01$

Exercise 3 (Hypothesis testing).
You are given a coin and want to find out if the coin is fair. In order to do so, you construct a hypothesis test with

$$H_0 : p = 0.5$$
$$\text{and } H_1 : p \neq 0.5.$$

Now you toss the coin $n = 10$ times and decide to reject H_0 if the number of heads is less than 3 or more than 7.

a) Compute the probability of the Type-I-error assuming that the null hypothesis is true.

b) Compute the probability of the Type-II-error if the true parameter is $p = 0.7$.