Winter term 2024/25
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Presence sheet 12

Mathematics for Machine Learning

Tutorial of **Week 13 (20.01. - 24.01.2024)**

Exercise 1 (Marginal distributions). Find an example of (or explain why it cannot exist)

a) Two different multivariate distributions that have the same marginal distributions.

b) Two different multivariate distributions, with independent components, that have the same marginal distributions.

Exercise 2 (Again marginal distributions). Consider the following experiment: In a bag, there are three balls, one red one, one green one and one blue one. You draw two balls without replacement, X is the color of the first one, Y the color of the second one. Determine the joint distribution (X, Y) and the marginal distributions of X and Y. Is the joint distribution the product of the marginals?

Exercise 3 (Conditional expectation). You roll two dice and consider their sum X.

a) What is the distribution of $\mathrm{E}(X \mid Y)$, where Y is the number of the first die?

b) What is the distribution of $\mathrm{E}(X \mid Y)$, where Y is 1 if one of the dice shows the number 1 and 0 otherwise?

c) What is the distribution of $\mathrm{E}(X \mid Y)$, where Y is the number of a third die?

Exercise 4 (Consistent estimator). Let $X_1, X_2, \dots, X_n$ be n independent and identically distributed random variables from a Bernoulli distribution with parameter p. That is

$$P(X_i = 1) = p, \quad P(X_i = 0) = 1 - p, \quad \text{for } i = 1, 2, \dots, n.$$

The task is to estimate the parameter p of the distribution using the sample data. We consider the estimator

$$\hat{p} = \frac{1}{n} \sum_{i=1}^{n} X_i.$$

a) What is the parameter space Θ? What is the statistical model $\mathcal{F}$?

b) Show that the estimator for $\hat{p}$ is unbiased.

c) Compute the variance of the estimator $\hat{p}$. Can you find an example of an estimator with zero variance?

d) Is the estimator $\hat{p}$ (strongly) consistent?