Winter term 2024/25
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Presence sheet 07

Mathematics for Machine Learning

Tutorial of **Week 08 (02.12. - 06.12.2024)**

Exercise 1 (Sublevel sets).
For a function f the sublevel sets are defined as $S_a := \{x \mid f(x) \leq a\}$. Prove that, if f is convex, then all sublevel sets are convex.

Exercise 2 (Convexity).
Decide if the following functions are convex, strictly convex or strongly convex?

a) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = e^x$

b) $f : (0,1) \to \mathbb{R}$ with $f(x) = x^3$

c) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2$

d) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x$

e) $f : \mathbb{R}^n \to \mathbb{R}$ with $f(x) = \|x\|_2$

Exercise 3 (Lagrangian).
Consider the following optimization problem:

$$\begin{aligned} \min_{x,y\in\mathbb{R}} \quad & f(x,y) = x^2 - y^2, \\ \text{subject to:} \quad & g(x,y) = x + y - 1 = 0. \end{aligned}$$

a) Is the problem convex?

b) Sketch the set of feasible points.

Exercise 4 (Convex functions).
Decide whether the following statements are true. Prove them or find a counterexample.

a) The product of two convex functions is convex.

b) The sum of two convex functions is convex.

c) The sum of a convex and strictly convex function is strictly convex.

d) The composition of two convex functions is convex.

e) The maximum of convex functions is convex.

f) The set of all global minimizers of a convex function is convex.