Winter term 2024/25
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Presence sheet 05

Mathematics for Machine Learning

Tutorial of **Week 06 (18.11. - 22.11.2024)**

Exercise 1 (Convergence).
Consider the sequence $(a_n)_{n\in\mathbb{N}} \subset [-2,2]$ with $a_n = (-1)^n \left(1+\frac{1}{n}\right)$

a) Is it convergent?

b) Find all accumulation points.

Exercise 2 (Convergence).
Find a sequence $(a_n)_{n\in\mathbb{N}} \subseteq I \subseteq \mathbb{R}$ with $a_n \in \mathbb{R}$, such that

a) it is a Cauchy sequence but does not converge on I.

b) it has exactly 3 accumulation points.

c) it has exactly $m \in \mathbb{N}$ accumulation points.

d) it has exactly one accumulation point but does not converge.

Exercise 3 (Convergence).
Decide if the following conjecture is true. If yes, prove it, if no, find counterexamples.
Conjecture: For every sequence $(a_n)_{n\in\mathbb{N}}$ with $a_n \in \mathbb{R}$ it holds that

$$\lim_{n\to\infty} \frac{a_n + \frac{1}{n}}{a_n} = \lim_{n\to\infty} \frac{a_n}{a_n} = 1.$$

Exercise 4. Consider a function $f : \mathbb{R} \to \mathbb{R}$ with $f(1) = 1$. Which of the following statements implies that there exists a point $x \in \mathbb{R}$ such that $f(x) = 0$.

a) f is continuous and $f(10) = -1$.

b) f is strictly monotonically decreasing and $f(10) = -1$.

Exercise 5 (Taylor Series).
Consider a function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = e^x$ and $a = 0$.

a) Calculate the Taylor series up do degree 3, that is, $T_3(x, a)$.

b) Calculate the Lagrangian remainder term $R_3(x, a)$ for some ξ that lies between a and x.